SECTION 12.1 EXERCISES

Review Questions

1. Give two pieces of information which, taken together, uniquely determine a plane.

2. Find a vector normal to the plane $-2x - 3y + 4z = 12$.

3. Where does the plane $-2x - 3y + 4z = 12$ intersect the coordinate axes?

4. What is an equation of the plane with a normal vector $\mathbf{n} = \langle 1, 1, 1 \rangle$ that passes through the point $(1, 0, 0)$?

5. To which coordinate axes are the following cylinders in $\mathbf{R}^3$ parallel: $x^2 + 2y^2 = 8$, $z^2 + 2y^2 = 8$, and $x^2 + 2z^2 = 8$?

6. Describe the graph of $x = z^2$ in $\mathbf{R}^3$.

7. What are the traces of a surface?

8. What is the name of the type of surface defined by the equation $y = \frac{x^2}{4} + \frac{z^2}{8}$?

9. What is the name of the type of surface defined by the equation $x^2 + \frac{y^2}{3} + 2z^2 = 1$?

10. What is the name of the type of surface defined by the equation $-y^2 - \frac{z^2}{2} + x^2 = 1$?

Basic Skills

11–14. Equations of planes *Find an equation of the plane that passes through the point P_0 with a normal vector* **n**.

11. $P_0(0, 2, -2)$; $\mathbf{n} = \langle 1, 1, -1 \rangle$

12. $P_0(1, 0, -3)$; $\mathbf{n} = \langle 1, -1, 2 \rangle$

13. $P_0(2, 3, 0)$; $\mathbf{n} = \langle -1, 2, -3 \rangle$

14. $P_0(1, 2, -3)$; $\mathbf{n} = \langle -1, 4, -3 \rangle$

15–18. Equations of planes *Find an equation of the following planes.*

15. The plane passing through the points $(1, 0, 3)$, $(0, 4, 2)$, and $(1, 1, 1)$

16. The plane passing through the points $(-1, 1, 1)$, $(0, 0, 2)$, and $(3, -1, -2)$

17. The plane passing through the points $(2, -1, 4)$, $(1, 1, -1)$, and $(-4, 1, 1)$

18. The plane passing through the points $(5, 3, 1)$, $(1, 3, -5)$, and $(-1, 3, 1)$

19–22. Properties of planes *Find the points at which the following planes intersect the coordinate axes and find equations of the lines where the planes intersect the coordinate planes. Sketch a graph of the plane.*

19. $3x - 2y + z = 6$

20. $-4x + 8z = 16$

21. $x + 3y - 5z - 30 = 0$

22. $12x - 9y + 4z + 72 = 0$

23–24. Equations of planes *For the following sets of planes, determine which pairs of planes in the set are parallel, orthogonal, or identical.*

23. $Q: 3x - 2y + z = 12$; $R: -x + 2y/3 - z/3 = 0$; $S: -x + 2y + 7z = 1$; $T: 3x/2 - y + z/2 = 6$

24. $Q: x + y - z = 0$; $R: y + z = 0$; $S: x - y = 0$; $T: x + y + z = 0$

25–28. Parallel planes *Find an equation of the plane parallel to the plane Q passing through the point P_0.*

25. $Q: -x + 2y - 4z = 1$; $P_0(1, 0, 4)$

26. $Q: 2x + y - z = 1$; $P_0(0, 2, -2)$

27. $Q: 4x + 3y - 2z = 12$; $P_0(1, -1, 3)$

28. $Q: x - 5y - 2z = 1$; $P_0(1, 2, 0)$

29–32. Intersecting planes *Find an equation of the line where the planes Q and R intersect.*

29. $Q: -x + 2y + z = 1$; $R: x + y + z = 0$

30. $Q: x + 2y - z = 1$; $R: x + y + z = 1$

31. $Q: 2x - y + 3z - 1 = 0$; $R: -x + 3y + z - 4 = 0$

32. $Q: x - y - 2z = 1$; $R: x + y + z = -1$

33–36. Cylinders in $\mathbf{R}^3$ *Consider the following cylinders in $\mathbf{R}^3$.*

a. Identify the coordinate axis to which the cylinder is parallel.
b. Sketch the cylinder.

33. $y - x^3 = 0$

34. $x - 2z^2 = 0$

35. $z - \ln y = 0$

36. $x - 1/y = 0$

37–60. Quadric surfaces *Consider the following equations of quadric surfaces.*

a. Find the intercepts with the three coordinate axes, when they exist.
b. Find the equations of the xy-, xz-, and yz-traces, when they exist.
c. Sketch a graph of the surface.

Ellipsoids

37. $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$

38. $4x^2 + y^2 + \frac{z^2}{2} = 1$

39. $\frac{x^2}{3} + 3y^2 + \frac{z^2}{12} = 3$

40. $\frac{x^2}{6} + 24y^2 + \frac{z^2}{24} - 6 = 0$

Elliptic paraboloids

41. $x = y^2 + z^2$

42. $z = \frac{x^2}{4} + \frac{y^2}{9}$

43. $9x - 81y^2 - \frac{z^2}{4} = 0$

44. $2y - \frac{x^2}{8} - \frac{z^2}{18} = 0$